2.1 Scalar Quantities and Vector Quantities

Scalar Quantities and Vector Quantities

Key Ideas

- Scalars are numbers that do not include information about direction or orientation. They are numbers with or without units. Most physical quantities have units, however. Scalars may be positive or negative.
- Vectors have a magnitude, which is a positive number, usually with units, and an orientation or direction.
- A vector is generally represented as an arrow.

Learning Objectives

By the end of this section, you will be able to:

- describe the difference between vector and scalar quantities,
- identify the magnitude and direction of a vector, and
- explain the effect of multiplying a vector quantity by a scalar.

Prior to starting this course, you have learned a significant amount about numbers, including integers and real numbers as both pure numbers and numbers with units; going forward, these more types of numbers will typically be referred to as **scalars**. Many familiar physical quantities can be specified completely by giving a number and an appropriate unit. For example, “a class period lasts $50\ \text{minutes}$” or “the gas tank in my car holds $45\ \text{liters}$” or “the temperature of the room is $21\ ^{\circ}\text{C}$.” A physical quantity that can be specified completely in this manner is called a **scalar quantity**.

Like scalars, **vectors** also have a number that will usually have units. Vectors also have an orientation or direction, which scalars do not have. The orientation is where the vector arrow is pointing, such as north, up, down, southwest, etc. The "amount" of the vector quantity is called its **magnitude**, which has a positive value.

Let's look at a relatable example. Let's say your favorite restaurant is three blocks from your home. You tell this to a friend from out of town, but fail to mention which way to walk from your home. You have only given the **distance** of three blocks, which is a scalar quantity. If you had said, "two blocks west and 1 block north of my place," then you would have given an exact location. Figure 2.1 shows a representation of this situation. The **displacement** vector is represented by an arrow that starts at your home (represented by the tail) and ends at the restaurant (head of the vector) and indicates the straight line distance between the starting and ending points, even though you actually walked the three blocks indicated. The magnitude of the displacement will be less than three blocks. The displacement vector in the figure is labeled with a letter with a small arrow above the letter, $\vec{d}$.

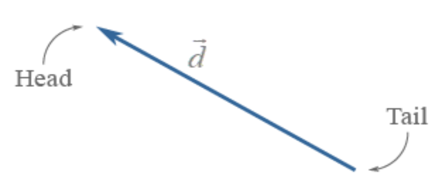


Figure 2.1 A vector is drawn from its tail to its head, which is at the tip of an arrowhead. The direction of a vector is indicated by the orientation of the arrow. The magnitude of a vector is proportional to its length, the distance between its head and tail.

If the vector represents a displacement, which means a change in position, then its magnitude will have units of length, but its length in a drawing is typically different than the length it represents. So, the length is scaled. For example, if the magnitude of the displacement were 2000m, we might represent that as a 4-cm long arrow and a 1000m displacement vector in the same drawing as a 2-cm long arrow. **Note**: when you are drawing a sketch of vectors in solving a problem, you usually do not have to exactly scale the lengths of your arrows, but should try to represent vectors of greater magnitude as longer than vectors that have smaller magnitude.

Scalar Multiplication

The magnitude of a vector can be changed by multiplying it by a number, a scalar quantity. If the scalar has a positive value, the direction is unchanged. If the multiplication is by a negative scalar, the direction of the vector is reversed.

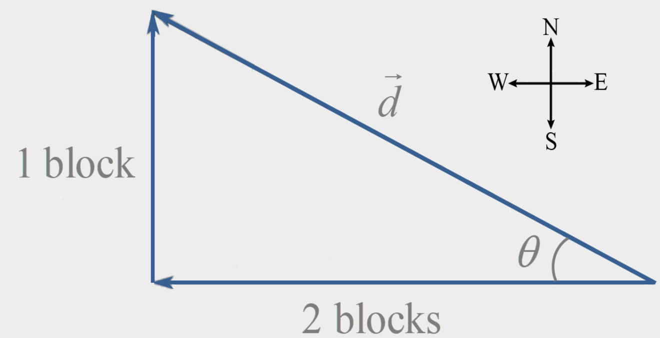
Examples:

- If $\vec{a} = (50\text{ m, to the left})$, then $5\vec{a} = (250\text{ m, to the left})$
- If $\vec{b} = (1500\text{ m, east})$, then $-2\vec{b} = (3000\text{ m, west})$

Note: the magnitude of a vector is always a positive number. Multiplying by a negative number reverses the direction of the vector.

Adding Vectors

At the beginning of this section, we introduced a displacement vector that resulted from a walk that started at your home and went 2 blocks to the west, then 1 block toward the north. What we were describing was how we can add two vectors to give a third vector. The figure below represents this journey that was actually taken using two vectors that are perpendicular to each other, one directed toward the west and one directed toward the north. The displacement vector is labeled $\vec{d}$ has its tail at your home and its head at the location of the final destination, your favorite restaurant. The three vectors form a right triangle and the displacement is the hypotenuse. We use the Pythagorean theorem to find its magnitude.



$d = \sqrt{(2\text{ blocks})^2 + (1\text{ block})^2} = 2.24\text{blocks}$

and the direction of the displacement can be found using the definition of the tangent and taking its inverse.

$\theta = \tan^{-1}\left(\frac{1\text{block}}{2\text{blocks}}\right) = 26.6^{\circ}$

Therefore, the displacement is

$\vec{d} = 2.24\text{blocks},\ 26.6^{\circ}$ north of west.